Impedance, Resonance, and Filters

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VO1NO

A Quick Review…

- Before discussing Impedance, we must first understand capacitive and inductive reactance.

Reactance

- **Reactance** is the **opposition** to the **flow of Alternating Current (AC).**
- **Reactance** has **no effect** on the flow of **Direct Current (DC).**

Capacitive Reactance

- **Capacitive Reactance** is the **opposition** to the **flow of AC by capacitance.**
- As the **frequency of the AC increases, Capacitive Reactance decreases.**
- The **Symbol** for **Capacitive Reactance** is $\mathbf{X_C}$**.**
- $\mathbf{X_C}$ is expressed in **ohms.**
- Even though it is expressed in ohms, **power is not dissipated by Reactance!** Energy **stored in a capacitor** during **one part of the AC cycle** is simply **returned to the circuit** during the **next part of the cycle!**

Capacitive Reactance

$$X_C = \frac{1}{2 \pi f C}$$

- Where:

  F = frequency in Hertz

  C = capacitance in Farads

  π = 3.14

Inductive Reactance

- **Inductive Reactance** is the **opposition** to the **flow of current** in an **AC circuit** caused by an **inductor.**
- As the **frequency increases,** Inductive Reactance **also increases.**
- The **symbol** for **Inductive Reactance** is X_L**.**
- Even though it is expressed in ohms, **power is not dissipated by Reactance!** Energy **stored in an inductor's magnetic field** during **one part of the AC cycle** is simply **returned to the circuit** during the **next part of the cycle!**

Inductive Reactance

$$X_L = 2\pi f L$$

- Where:

  f = frequency in Hertz

  L = inductance in henrys

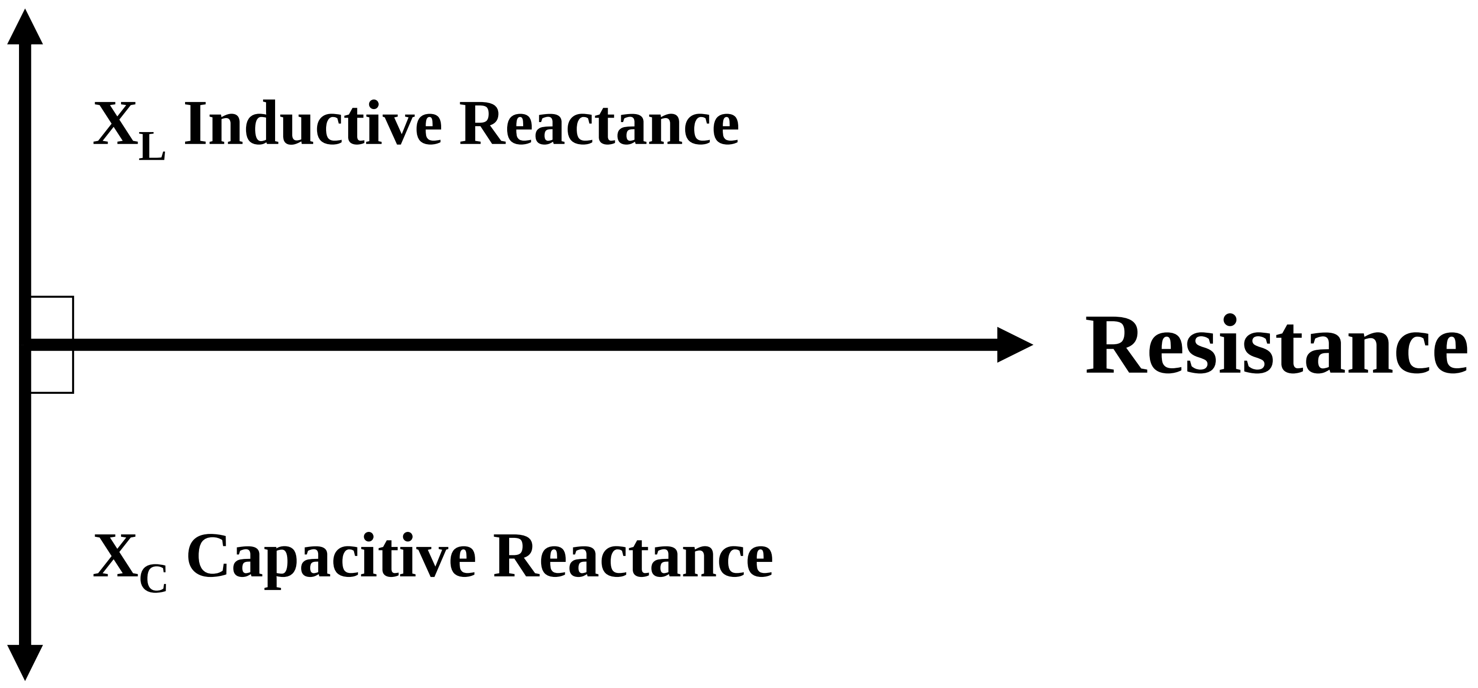
  π = 3.14

Current versus Voltage

- In a simple **resistive** circuit, the **current and voltage are always in phase.**
- For reasons beyond the scope of the Basic Course, the current and voltage are **not in phase in AC circuits** that contain **capacitance and/or inductance.**
- The **current** across a **capacitor leads the voltage by 90 degrees.**
- The **current** across an **inductor lags the voltage by 90 degrees.**

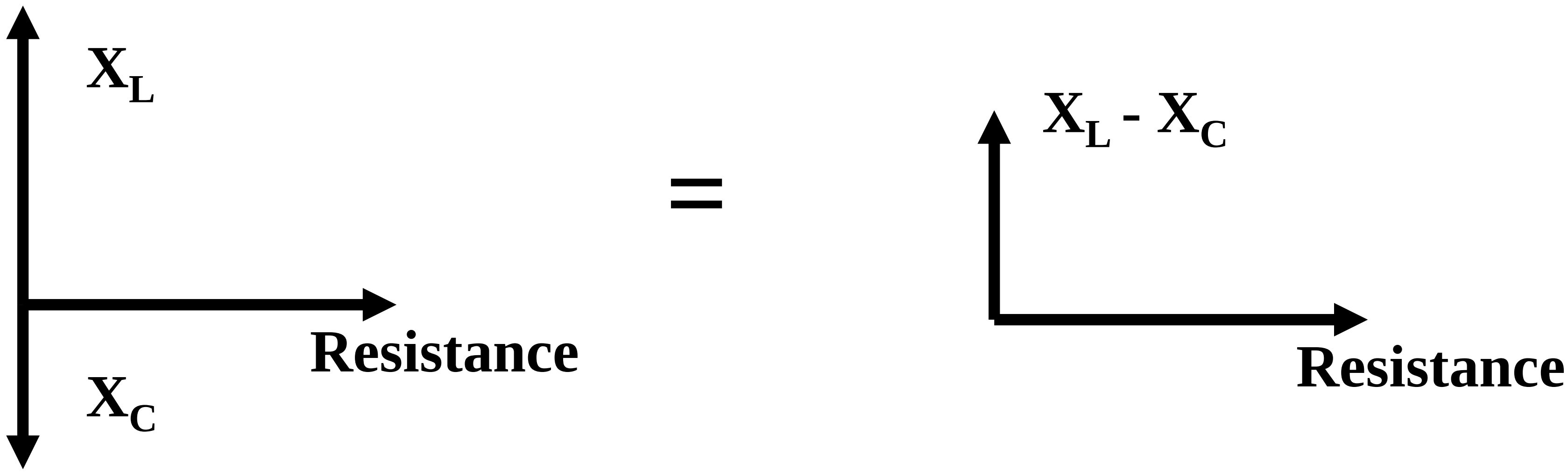
Vector Representation

- When plotted as **vectors, series circuits** containing Inductance, Capacitance and Resistance **(LCR)** can be represented as such:



Inductive vs Capacitive Reactance

- Inductive and Capacitive Reactance **cannot** be **added together** to give an overall reactance.
- In fact, they tend to **cancel each other out.**

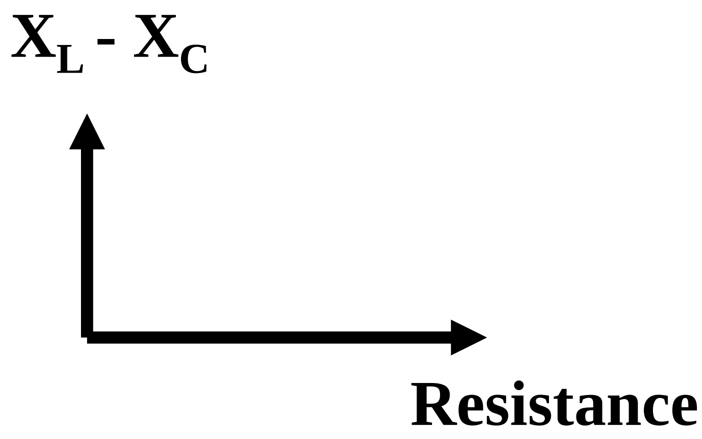


Impedance

- When a circuit contains both **resistance** and **reactance,** the **opposition to the flow of AC** is called **Impedance,** abbreviated **Z.**
- Because **Resistance** and **Reactance** are **not in phase** however, we must use **vectors** to determine the **Impedance,** even if Inductive and Capacitive Reactance have partly **cancelled each other out.**

Vector Addition

Through the use of **vector addition,** the Impedance can be determined…



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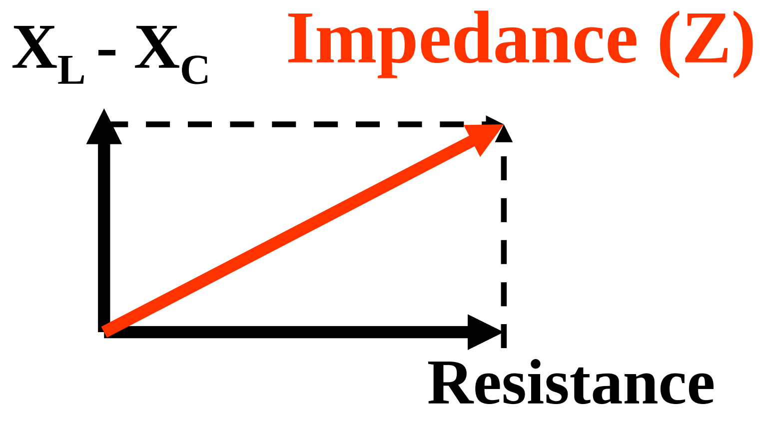

LCR Circuit Impedance Formula

- Rather than plot vectors every time we need to determine impedance however, we can use a **formula:**

$$\mathbf{Z} = \sqrt{R^2 + (X_L - X_c)^2}$$

- Note that because the difference between X_L and X_C is squared, **it doesn't matter** what term is subtracted from what – you can use $\mathbf{X_C - X_L}$ if that is more convenient.

LCR Circuit Impedance Example

- Resistance = 120 Ohms
- X_L = 40 Ohms
- X_C = 130 Ohms

- Z = Sqr Root [$R^2 + (X_C - X_L)^2$]

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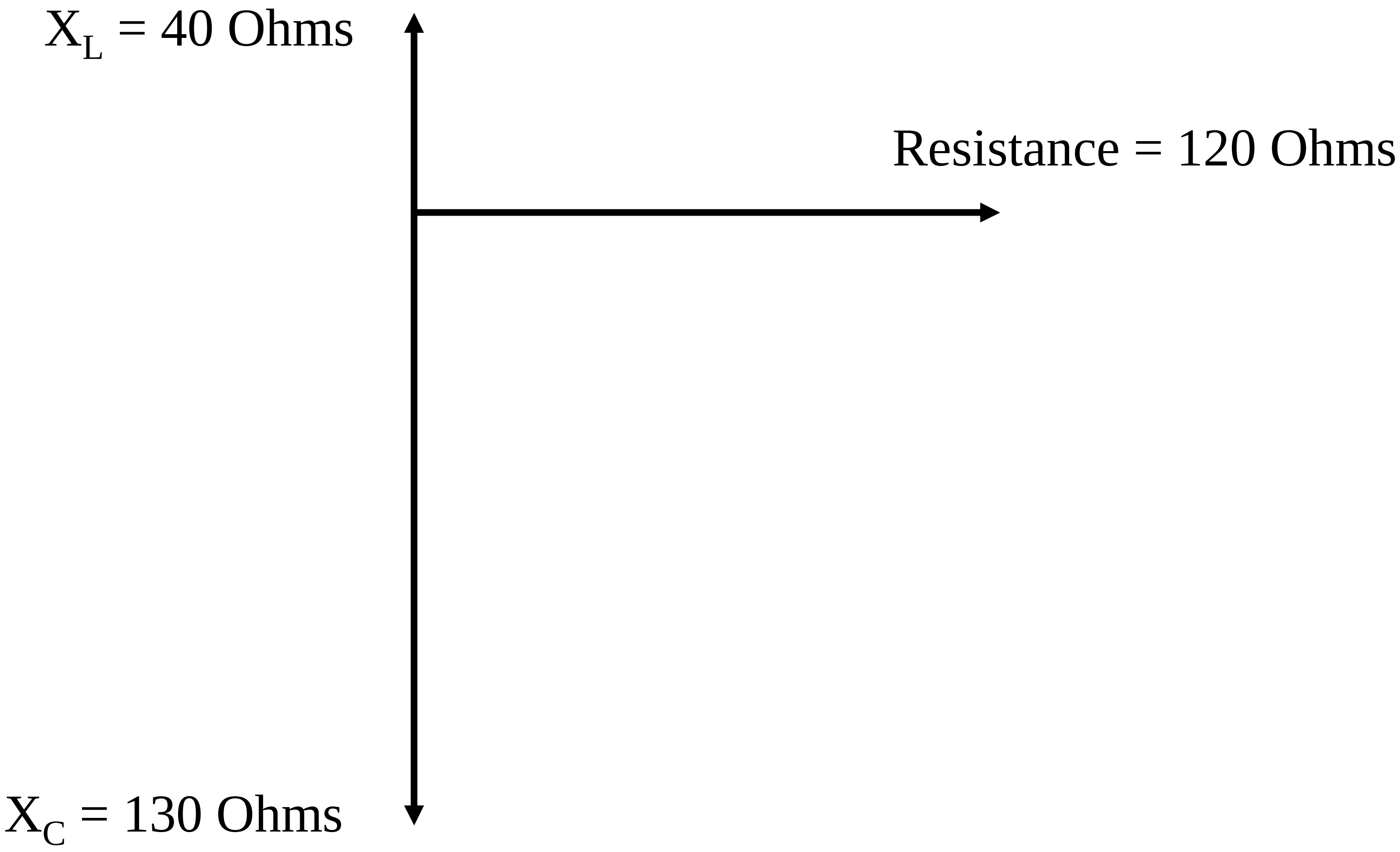
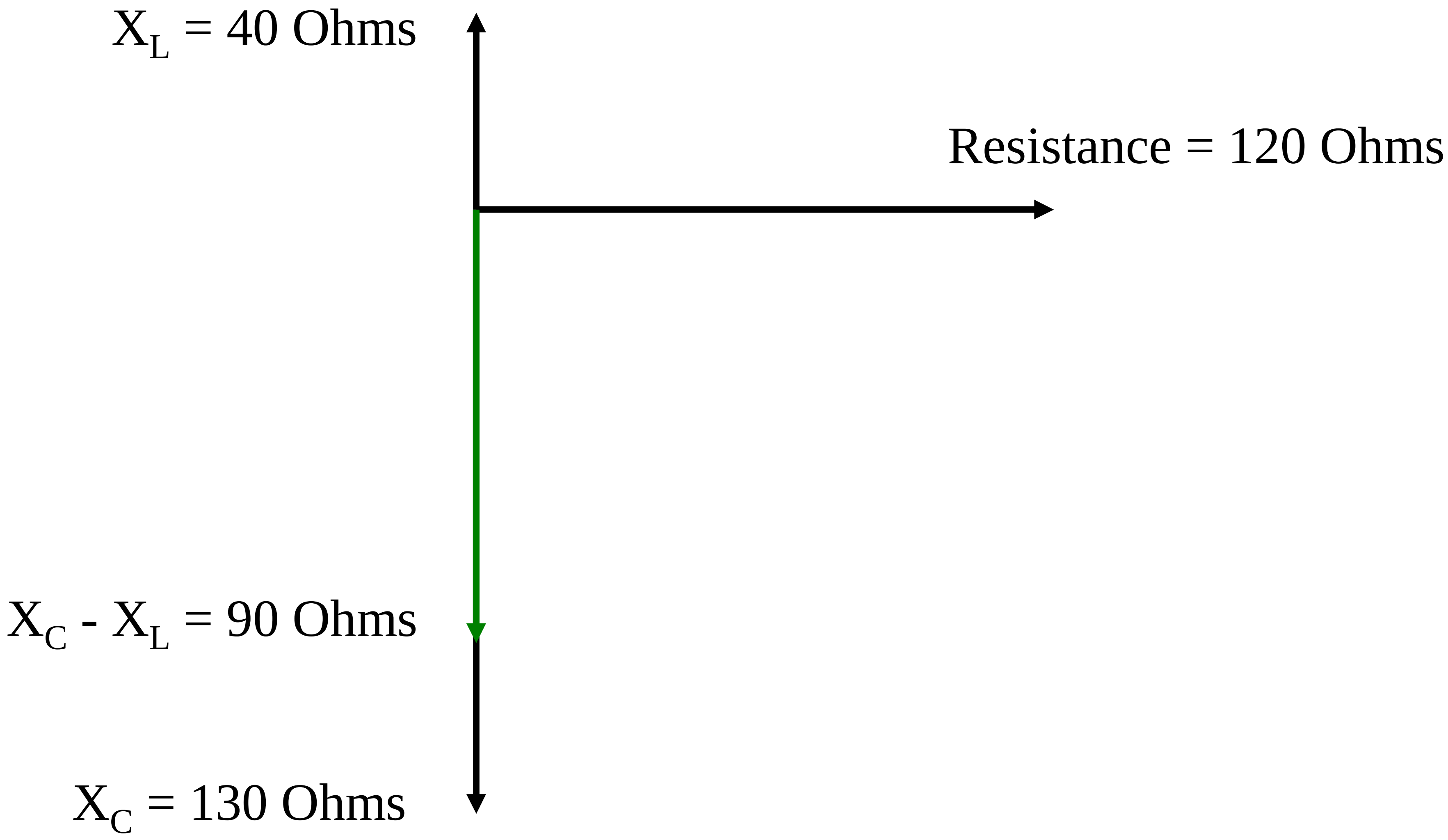
  = 150 Ohms

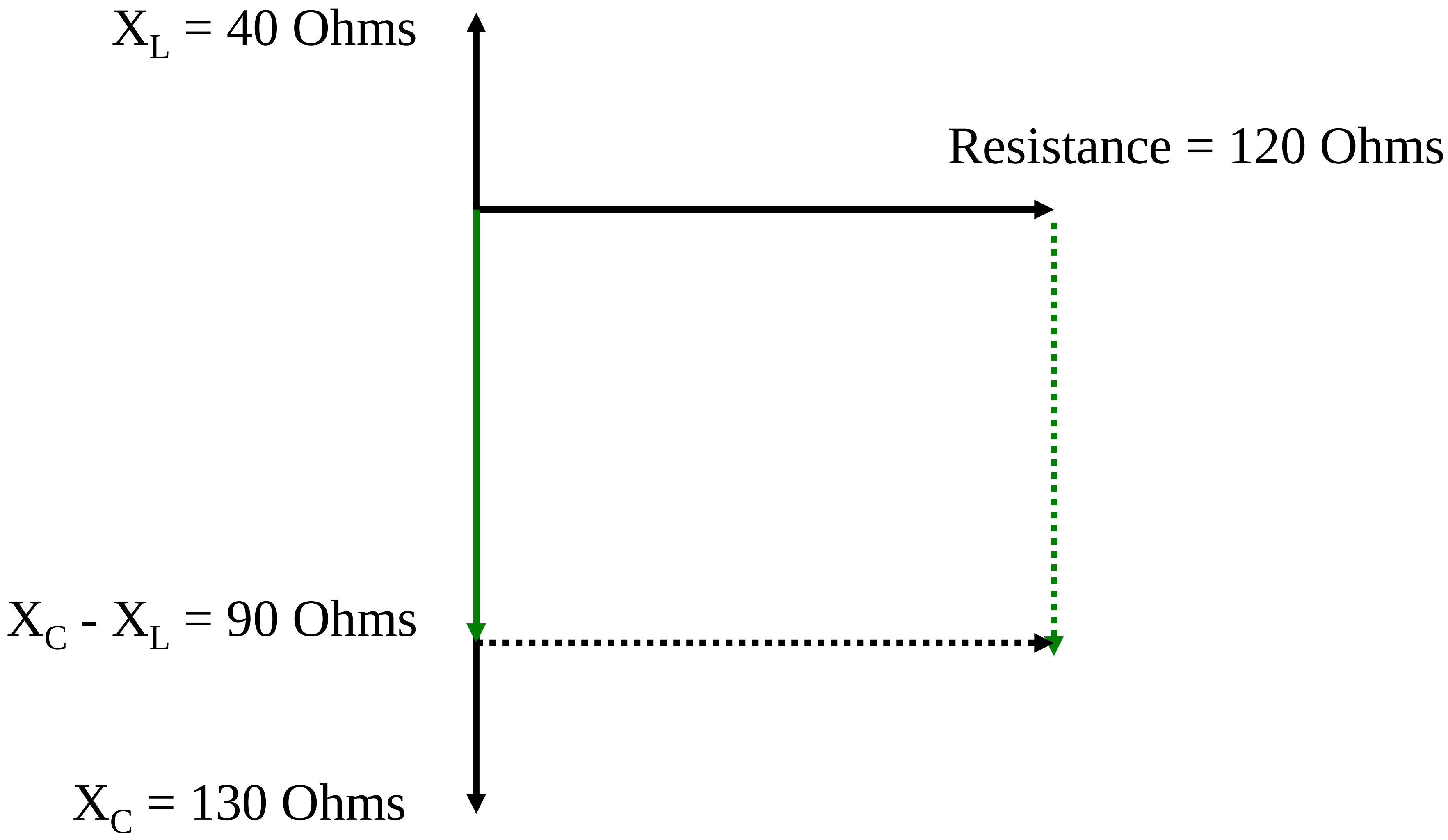
LCR Circuit Impedance Example



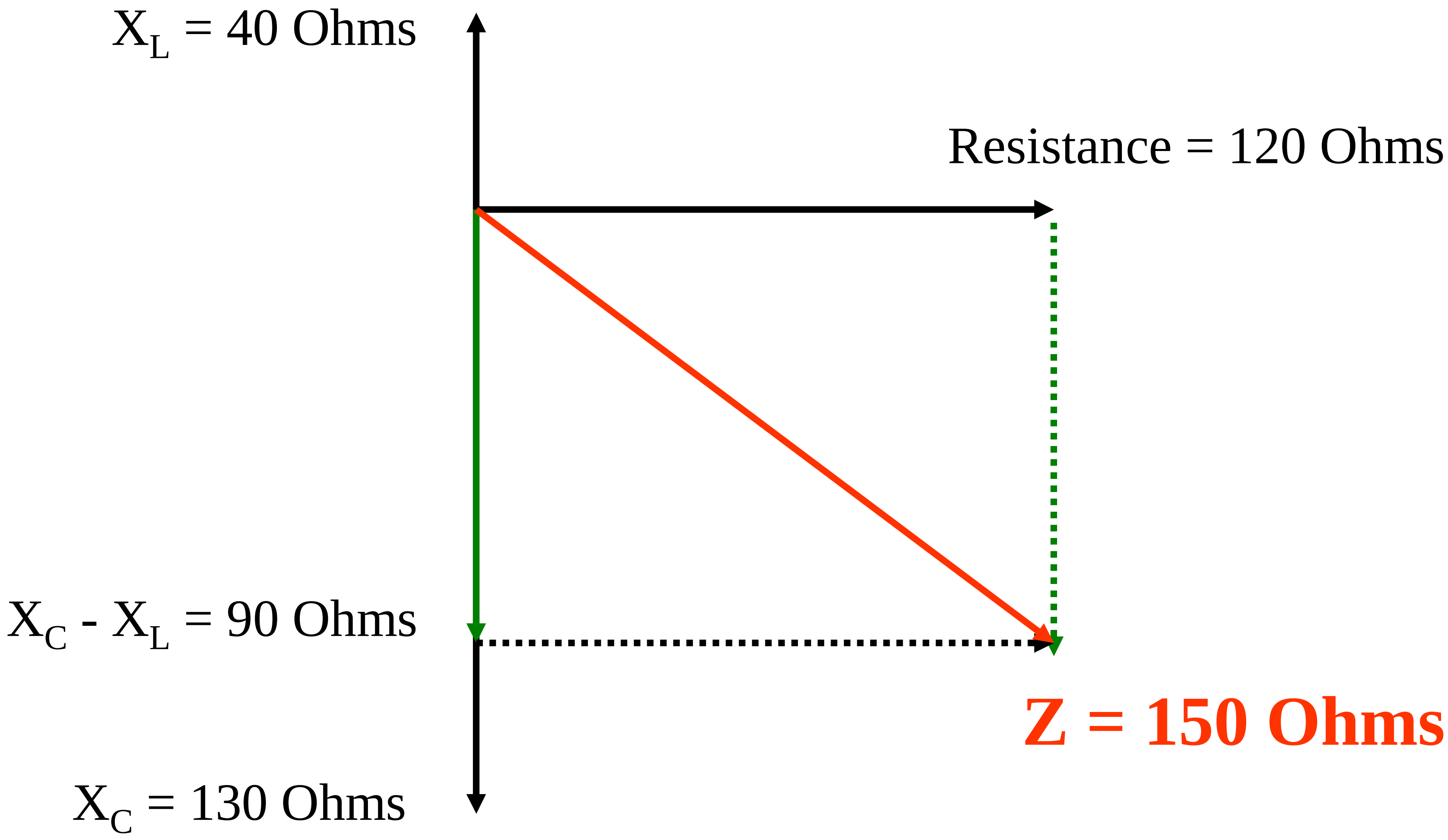
LCR Circuit Impedance Example



LCR Circuit Impedance Example



LCR Circuit Impedance Example



Impedance Matching

- Many electronic devices and circuits (speakers, microphones, antennas, transmission lines, amplifiers etc.) have their own **characteristic impedance.**
- When interconnecting these devices and circuits, **maximum power transfer** will take place if the various **impedances are matched.**

Matching with Transformers

- **Transformers** are often used to **match impedances** which are primarily **resistive.**
- This is especially true for **antenna and transmission line systems.**
- Take the matching transformer **(Balun – BALanced to UNbalanced)** used to match 300 Ohms to 75 Ohms in TV systems.

300 – 75 Ohm Balun

- These **matching transformers** are widely used for TV systems, and consist of a **small ferrite core** with **two windings.**
- In addition to the **impedance transformation,** it also converts between a **balanced** system and an **unbalanced** system.

Number of Turns

- Different impedances can be matched quite easily by adjusting the number of turns using the following formula:

$$\mathbf{Z_S / Z_P = N_S^2 / N_P^2}$$

Or…

$$\mathbf{N_S / N_P = \sqrt{Z_S / Z_P}}$$

Number of Turns - Example

- Z_S = 300 Ohms, Z_P = 75 Ohms

$$\mathbf{N_S / N_P = \sqrt{Z_S / Z_P}}$$

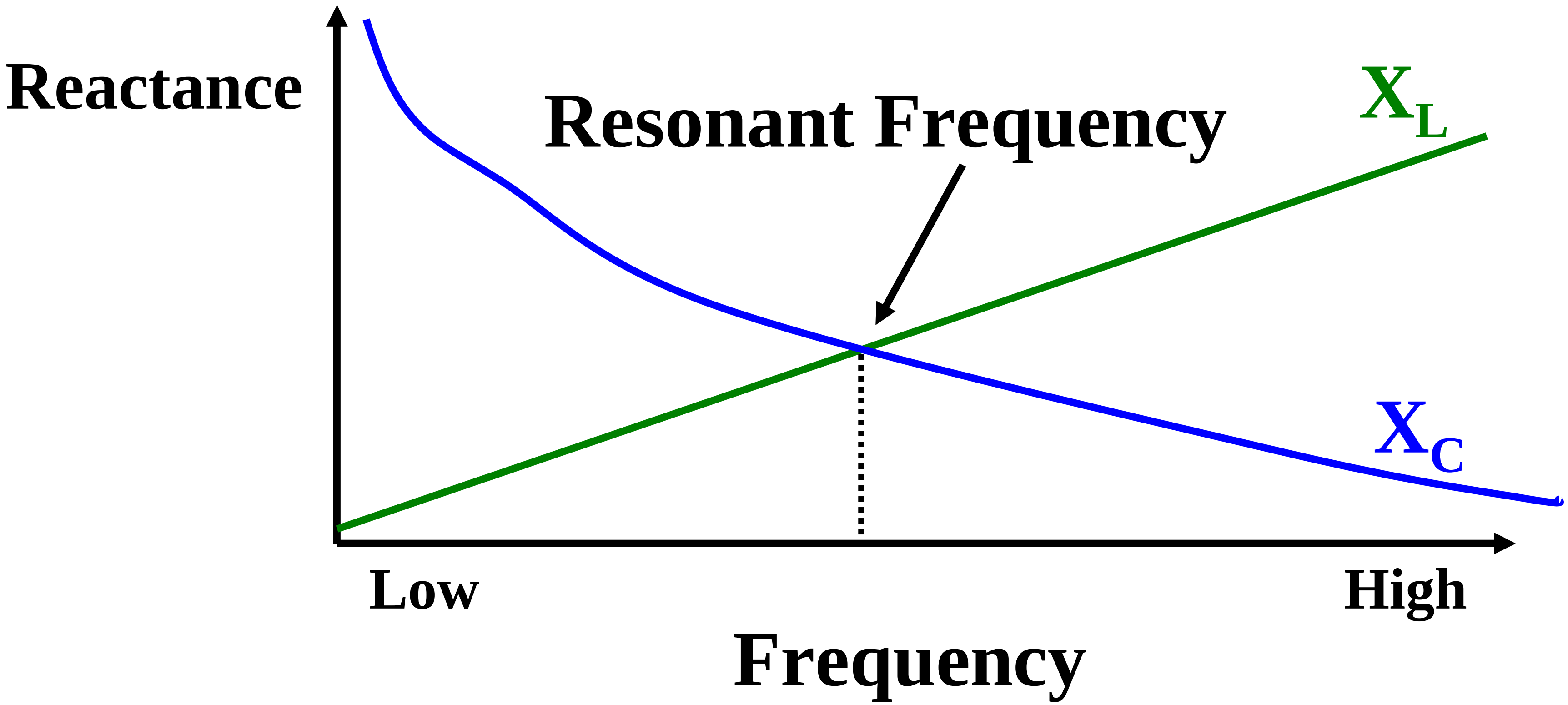
$$\mathbf{N_S / N_P = \sqrt{300 / 75} = \sqrt{4} = 2}$$

- The **turns ratio** is 2:1 ie: for **every turn** on the **primary** winding, there are **two** on the **secondary** winding.
- The actual number of turns depends on the core material.

Resonance

- In electronic circuits, a special condition exists when **Inductive** and **Capacitive Reactance** are **equal** to each other **($X_L = X_C$).**
- When that happens in Series LCR circuits, **X_L and X_C cancel** each other out, leaving only Resistance to oppose the flow of AC current.
- This condition is know as **Resonance,** and occurs at **only one frequency,** known as the **Resonant Frequency (F_R).**

Resonant Frequency



Resonant Frequency

- At Resonance, $\mathbf{X_C = X_L}$ so

$$X_C = \frac{1}{2 \pi f C} = X_L = 2 \pi f L$$

- With a little mathematical wizardry, we can rearrange that equation to determine the **Resonant Frequency** $\mathbf{F_R}$ as follows…

Resonant Frequency

$$F_R = \frac{1}{2\pi\sqrt{LC}}$$

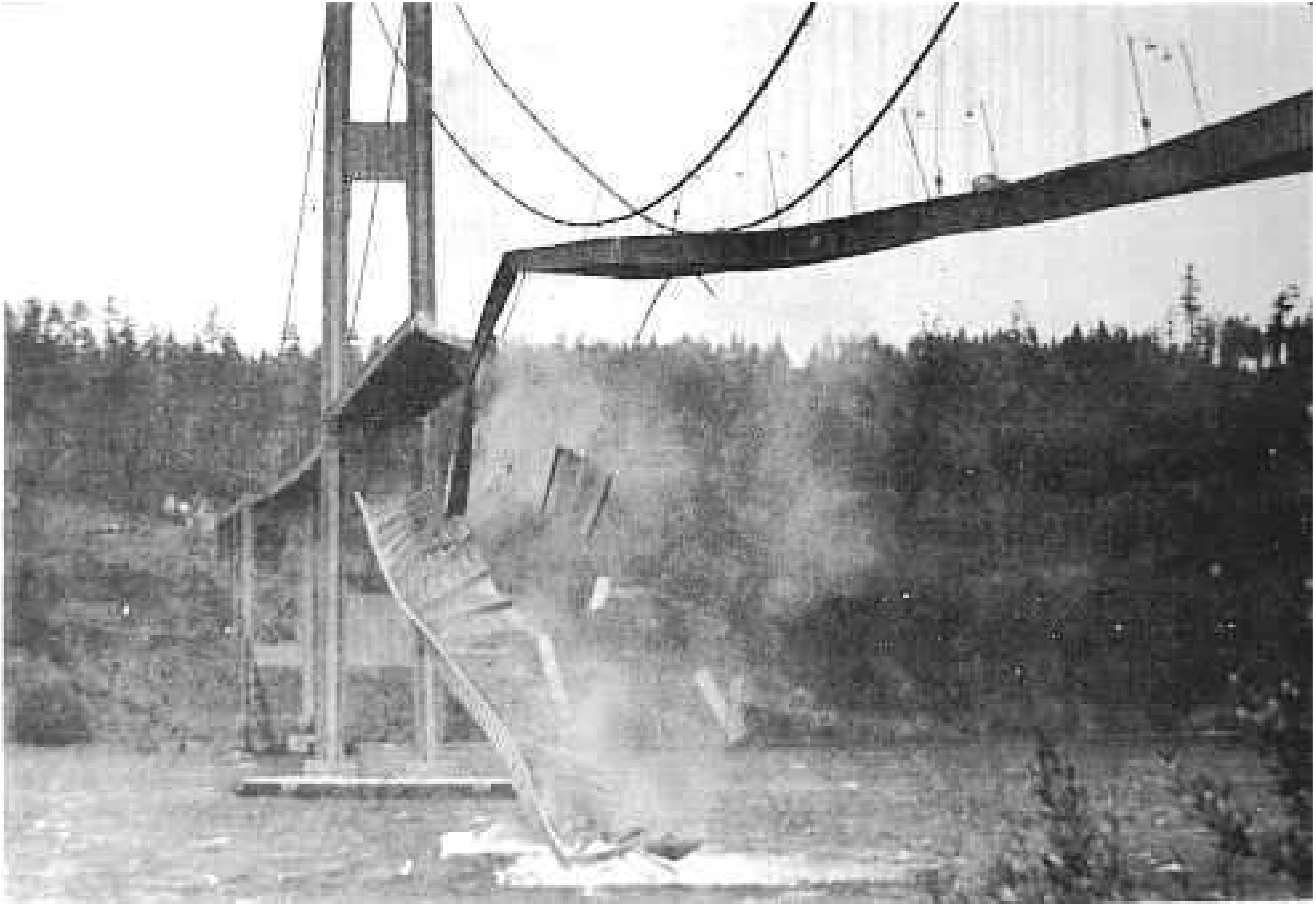
- Where:

  F_R = Resonant Frequency in Hertz

  L = Inductance in henrys

  C = Capacitance in Farads

Resonance is not always a good thing however…



Tacoma Narrows suspension bridge

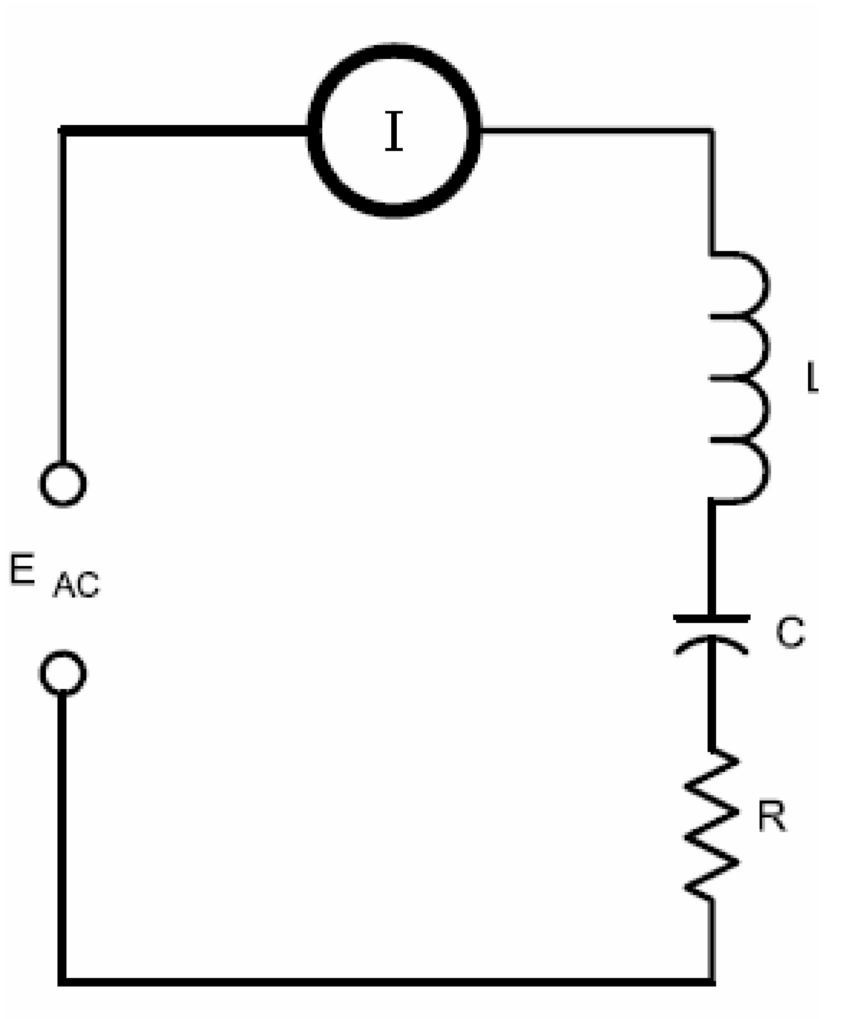
Tuned Circuits

- Circuits containing **Capacitors and Inductors** are often referred to as **Tuned Circuits.**
- They have **many uses** in electronics – every time you tune a radio, you are varying the **resonant frequency** of a **tuned circuit.**

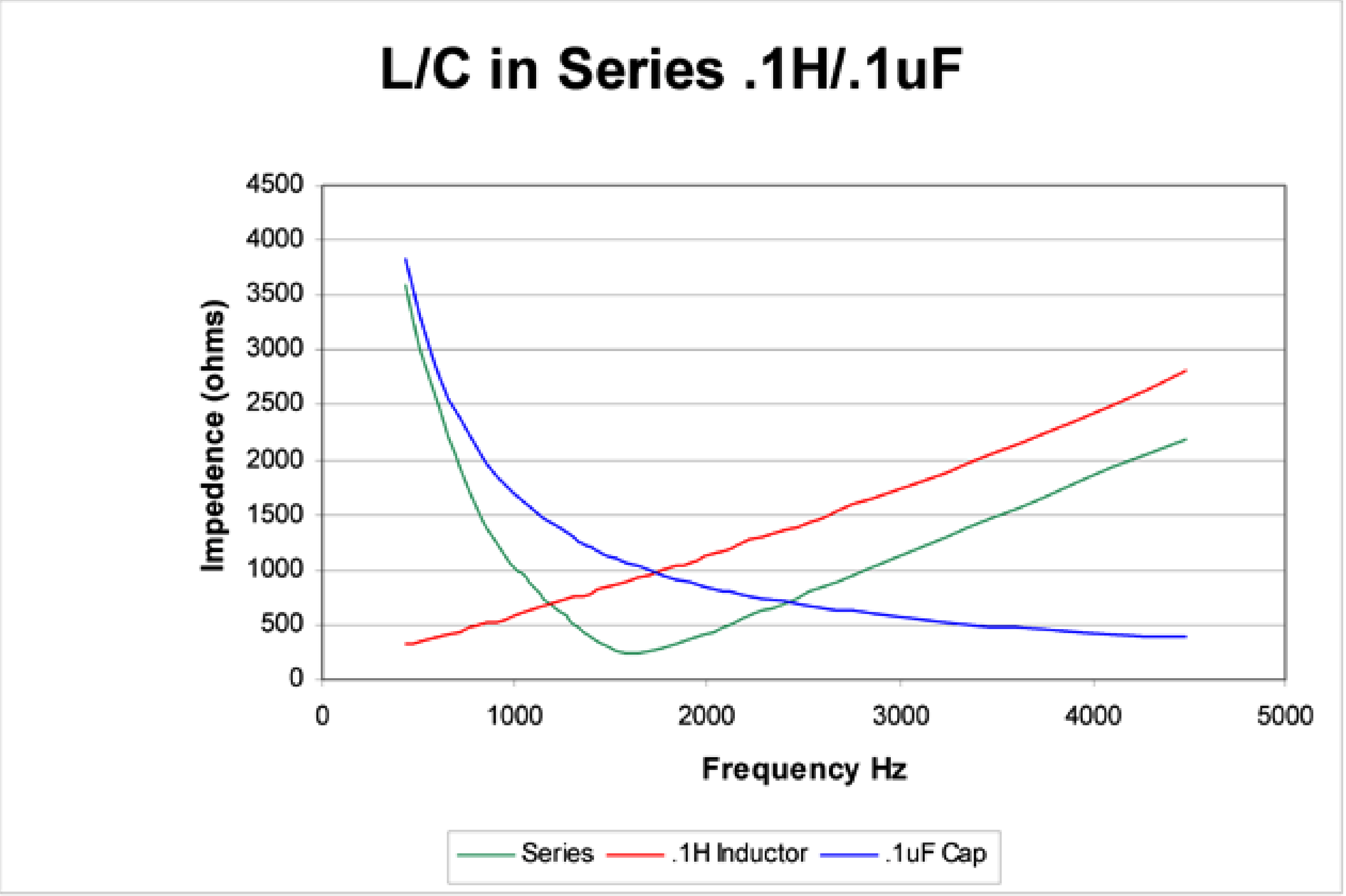
Series LCR Circuit

- When a **Series LCR** circuit is in **Resonance, current** in that circuit is at its **greatest** (the **Impedance** is at its **lowest**).
- There are **two ways** to achieve **Resonance** in **Series LCR** circuits:
  - **Vary the applied frequency** until we find the point where $X_C = X_L$.
  - Keep the frequency constant and **vary the value of the capacitance or inductance,** or both, until $X_C = X_L$.

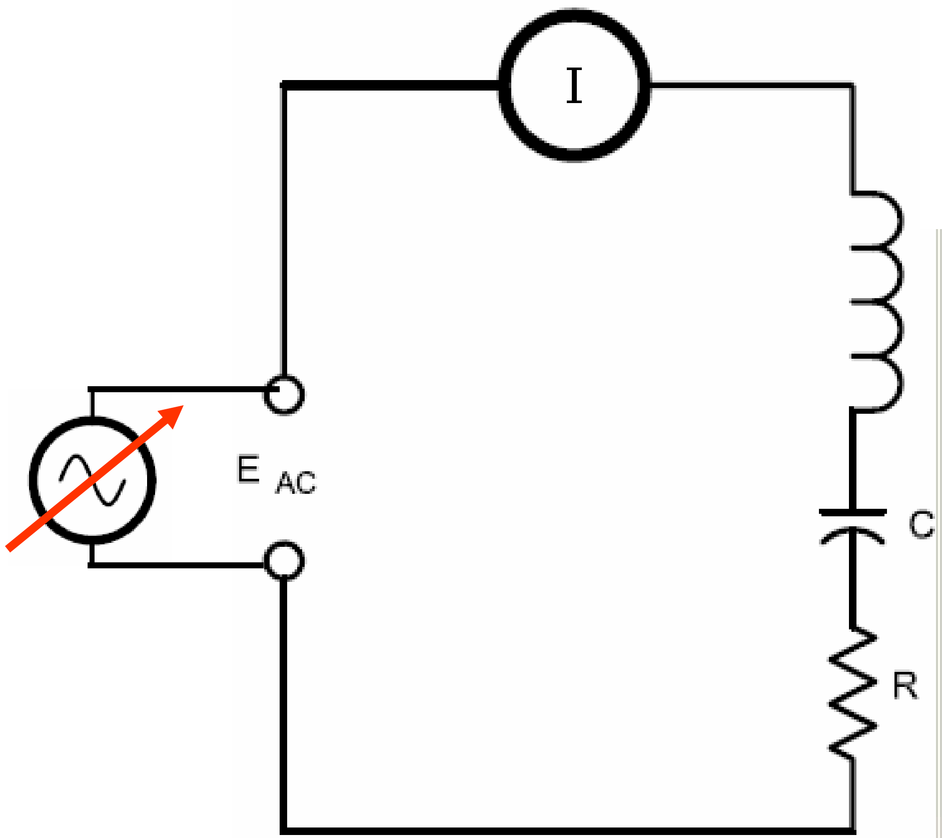
Series LCR Circuit

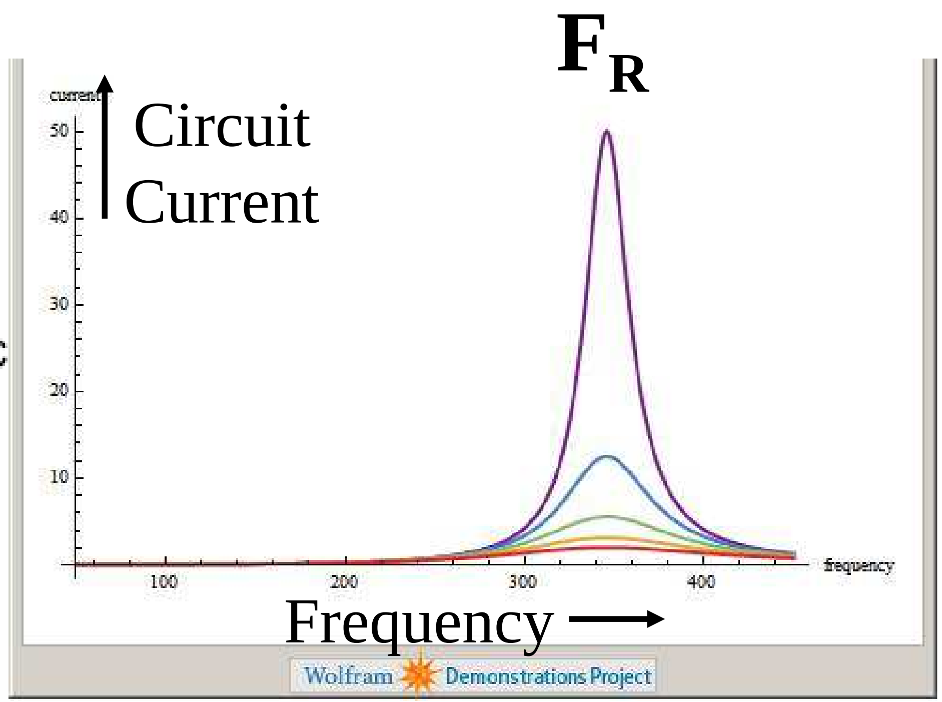


Series LCR Circuit Impedance

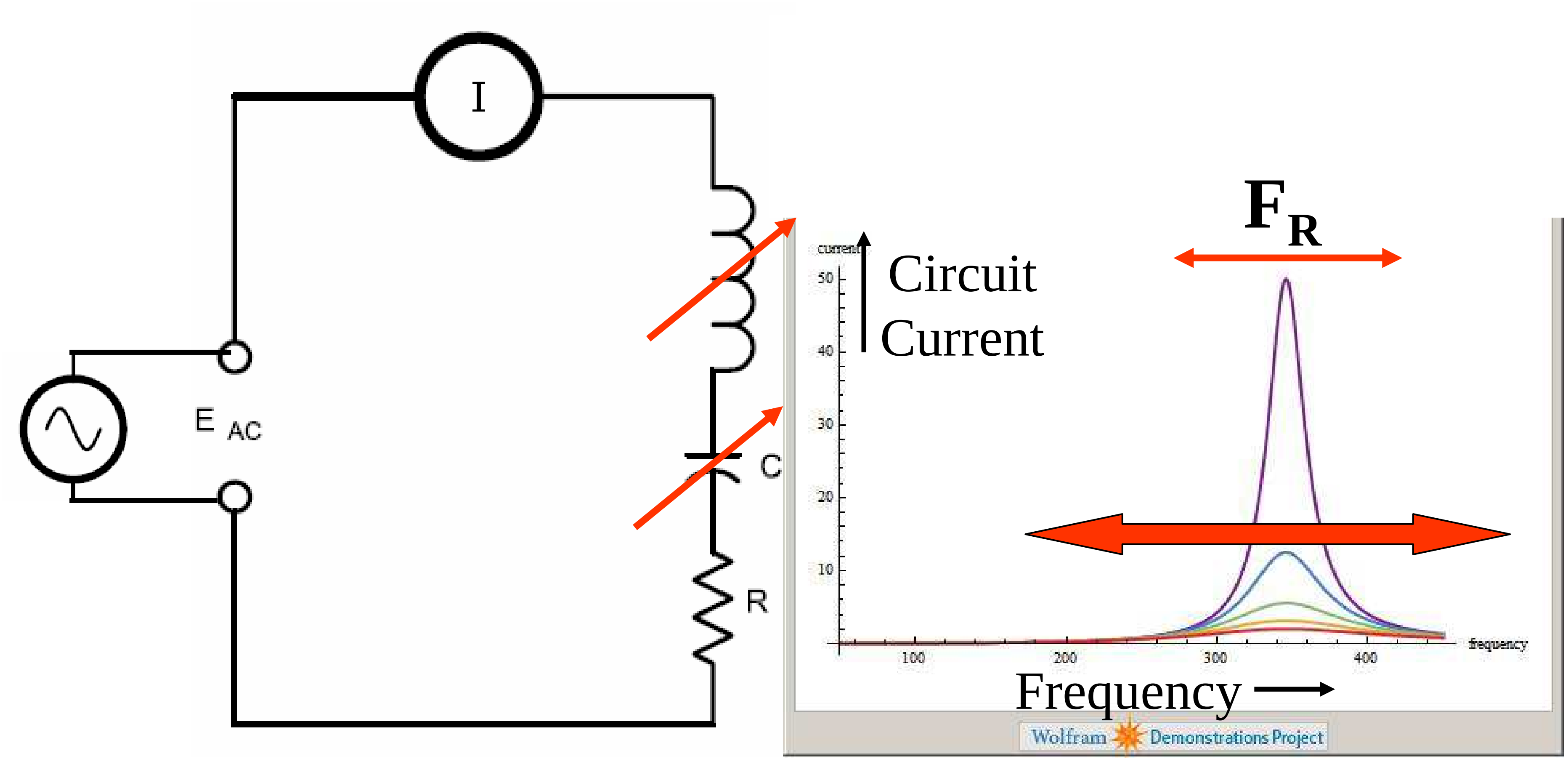


Varying the Frequency

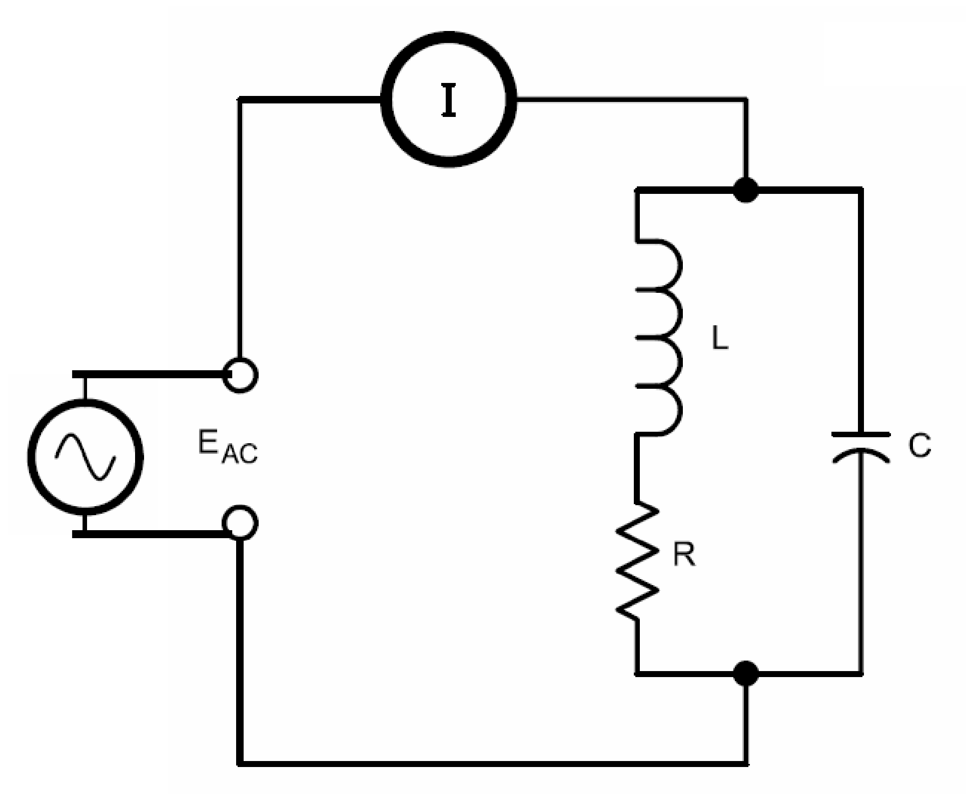




Varying Capacitance or Inductance



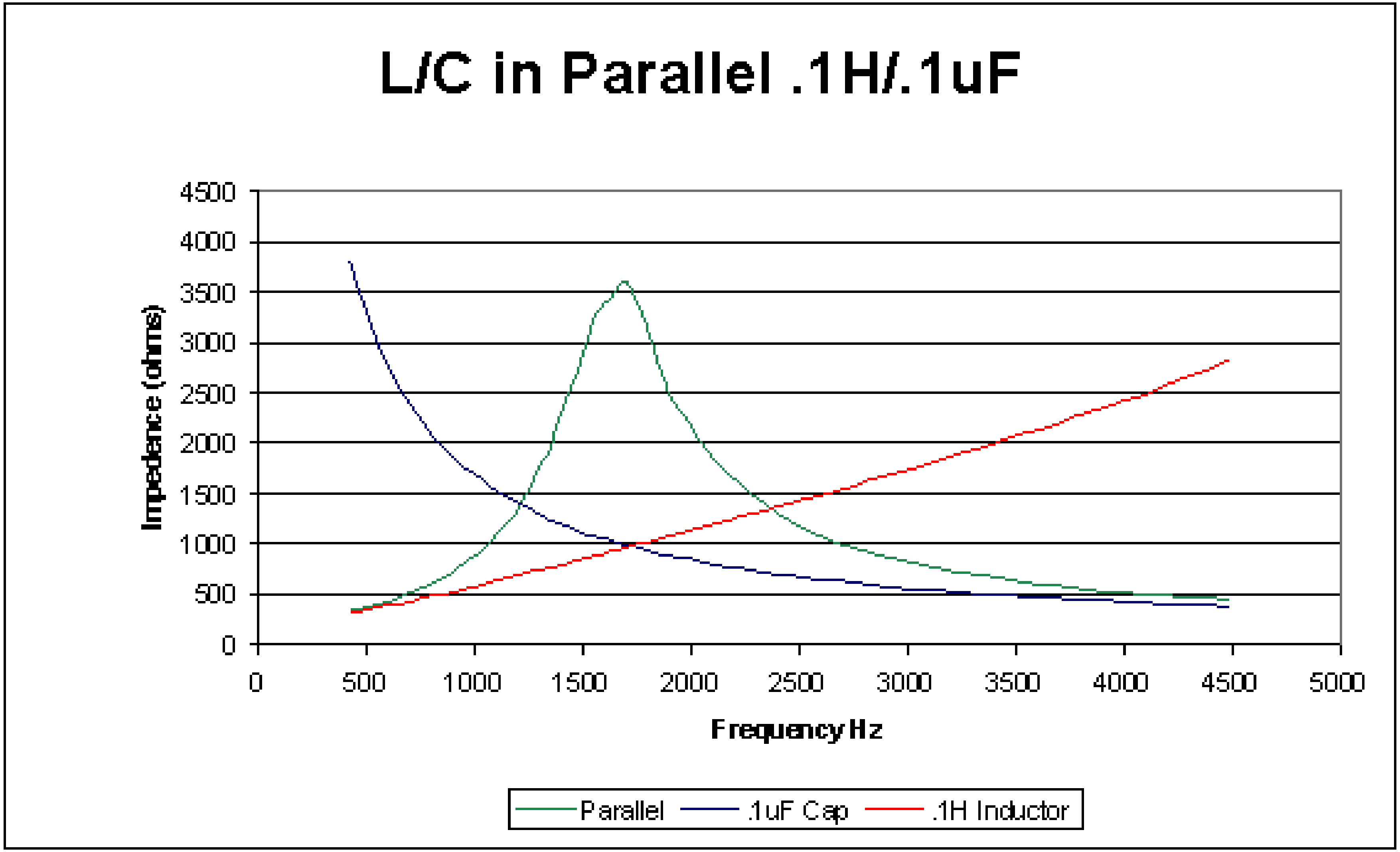
Parallel LCR Circuits



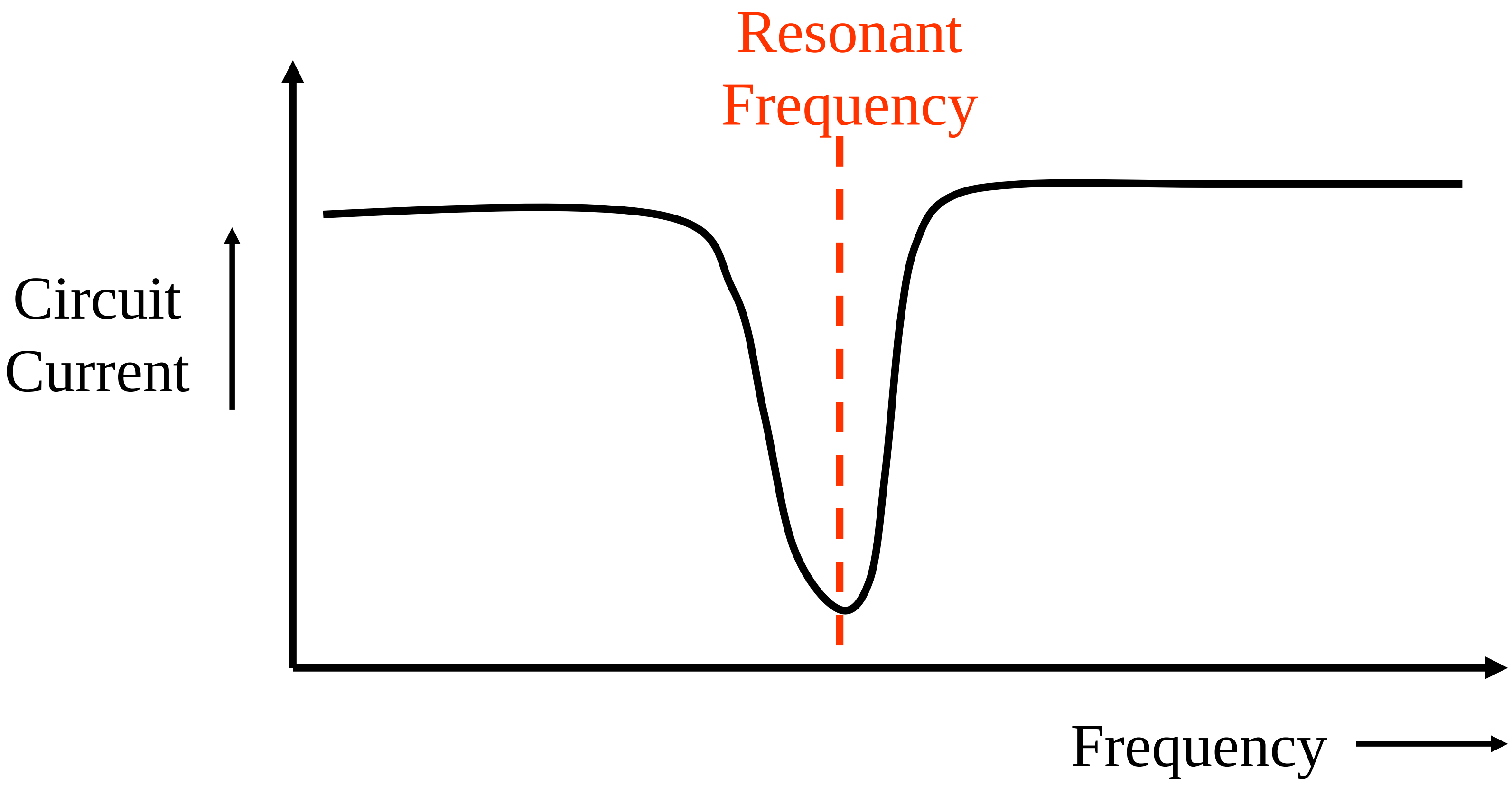
Parallel LCR Circuits

- In a **Parallel LCR Circuit**, the **current** is **lowest at Resonance** (the **impedance** is at its **highest).**
- Parallel LCR circuits are used to **reject a specific frequency** while allowing all others to pass.

Parallel LCR Circuit Impedance



Parallel LCR Circuit Current

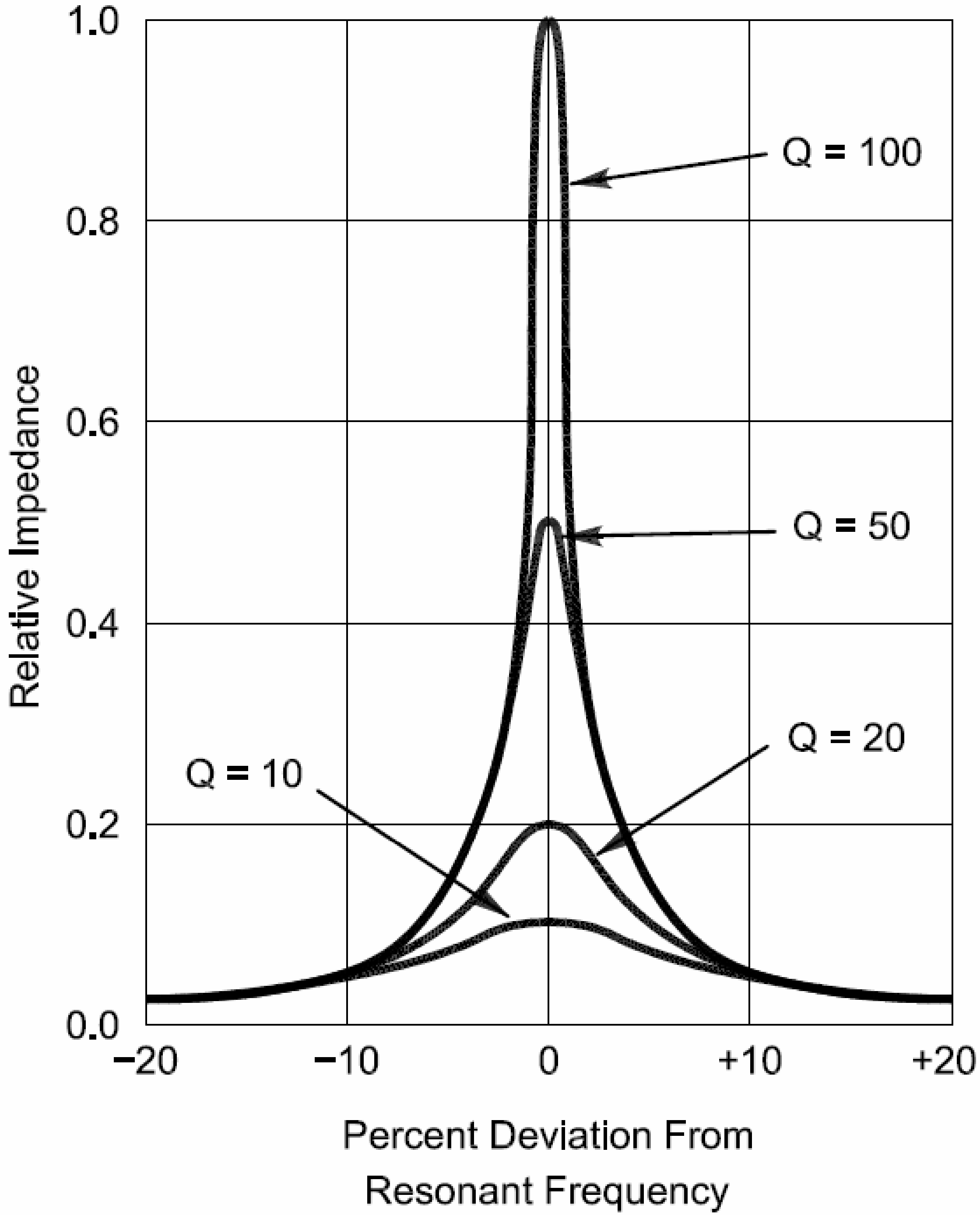


Circuit Quality

- In a **resonant series LCR circuit,** energy is stored **alternately** in the **electric field of the capacitor,** and then the **magnetic field of the inductor.**
- This causes a **current** to flow **between them.**
- Anything that **removes energy** from this circuit **broadens the range of frequencies** affected by the circuit, but **increases the impedance** at the **resonant frequency.**

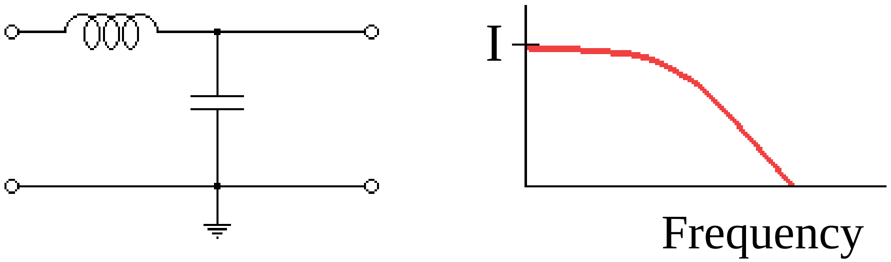
Circuit Quality

- The **“Q”, or Quality** of a series LCR circuit is defined as the **ratio** of either **X_C or X_L** to the **resistance** in the circuit.
- **Below** resonance **“Q” = X_C / R**
- **Above** resonance **“Q” = X_L / R**
- At resonance $X_C = X_L$
- A “Q” of **100** is **high**, while a “Q” of **10** is **low**.

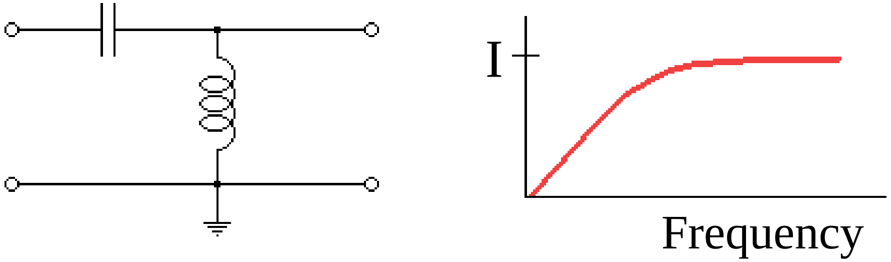

1.0
0.8
0.6
0.4
0.2
0.0
Relative Impedance
Q = 100
Q = 50
Q = 20
Q = 10
−20
−10
0
+10
+20
Percent Deviation From
Resonant Frequency

Filters

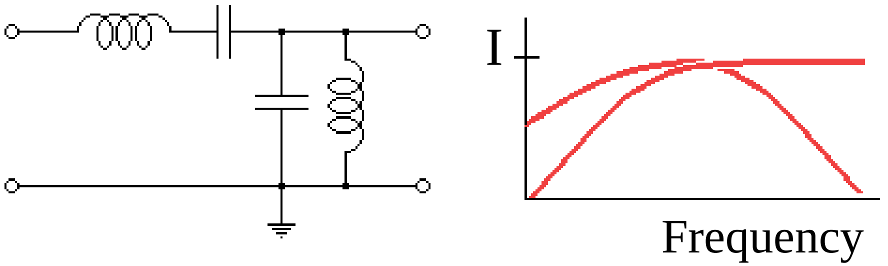
- By the proper selection of capacitors and inductors, it is possible to design **Filters** that can **pass desired frequencies**, and **reject unwanted frequencies.**



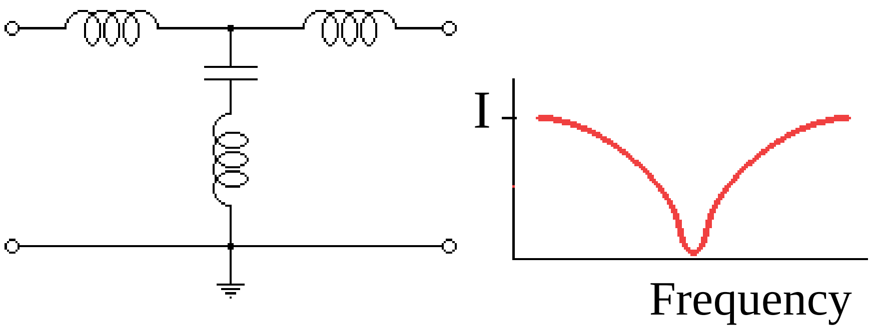
Low Pass Filter



High Pass Filter

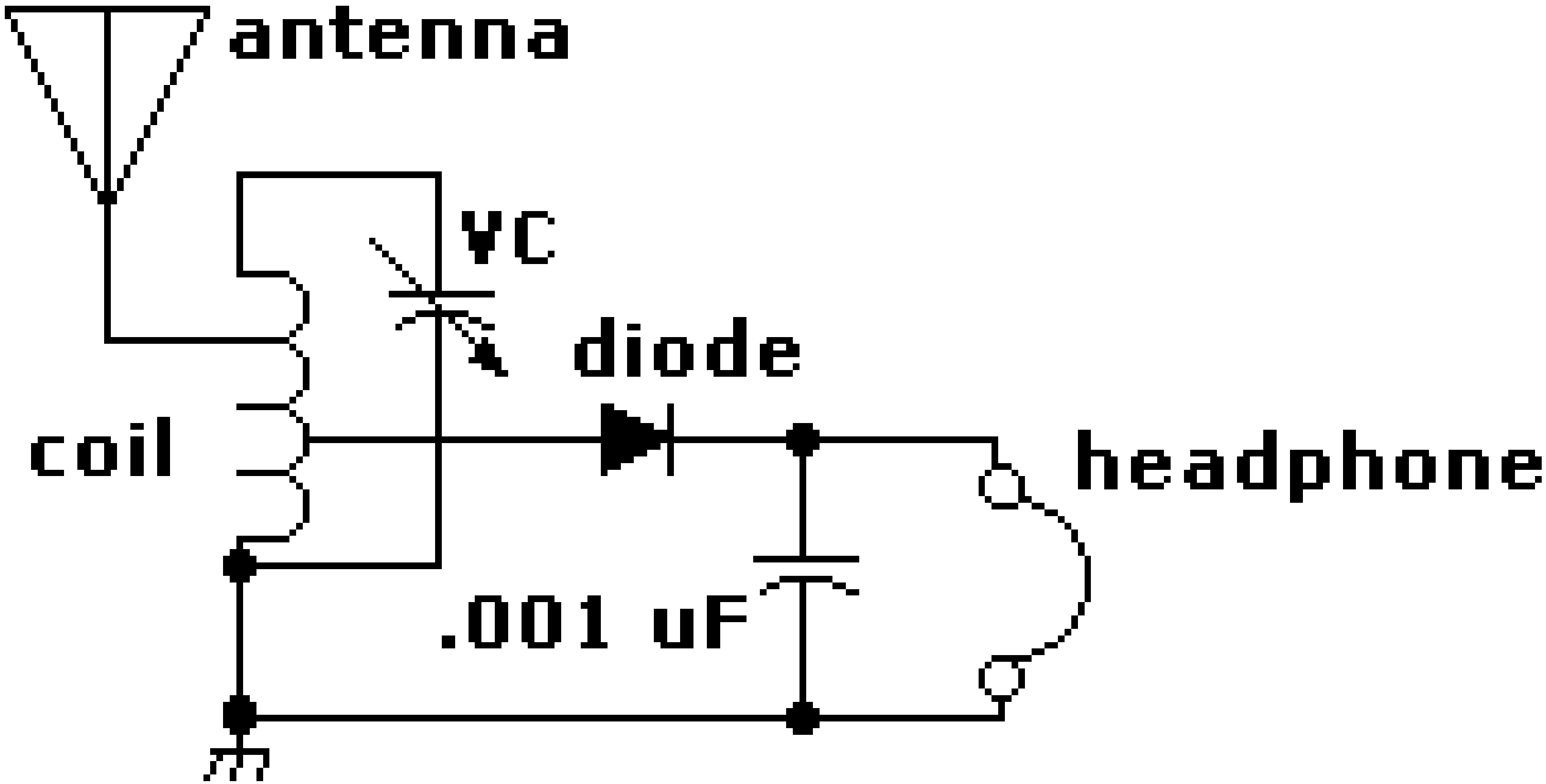


Band Pass Filter

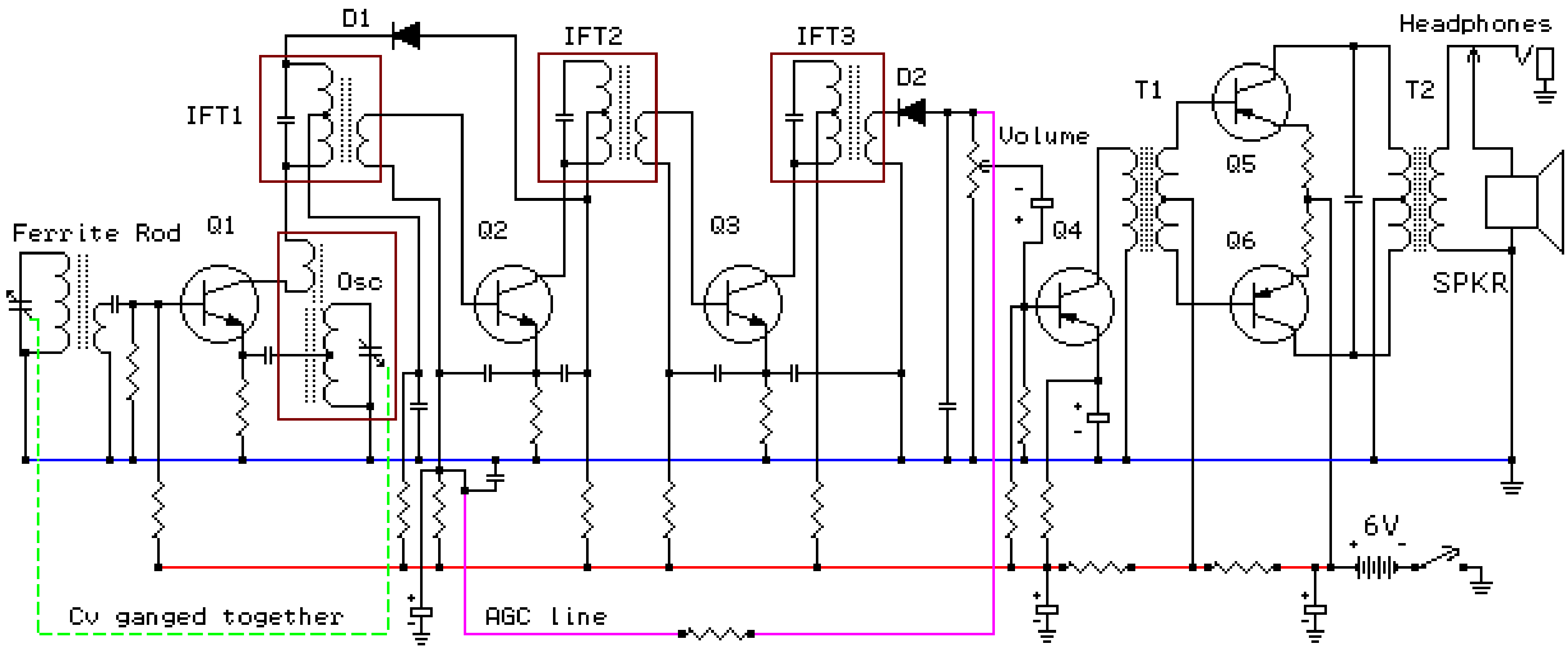


Notch Filter

Crystal Radio Tuned Circuit



6 Transistor Radio



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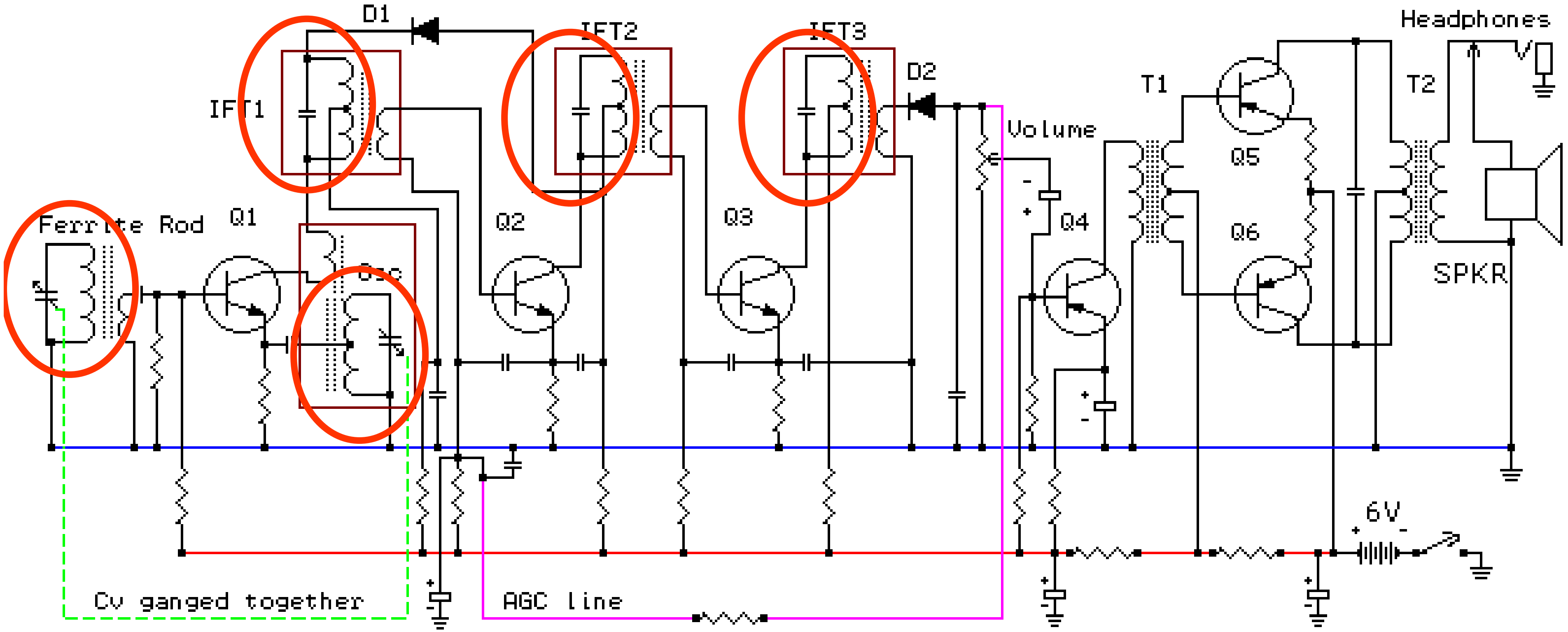
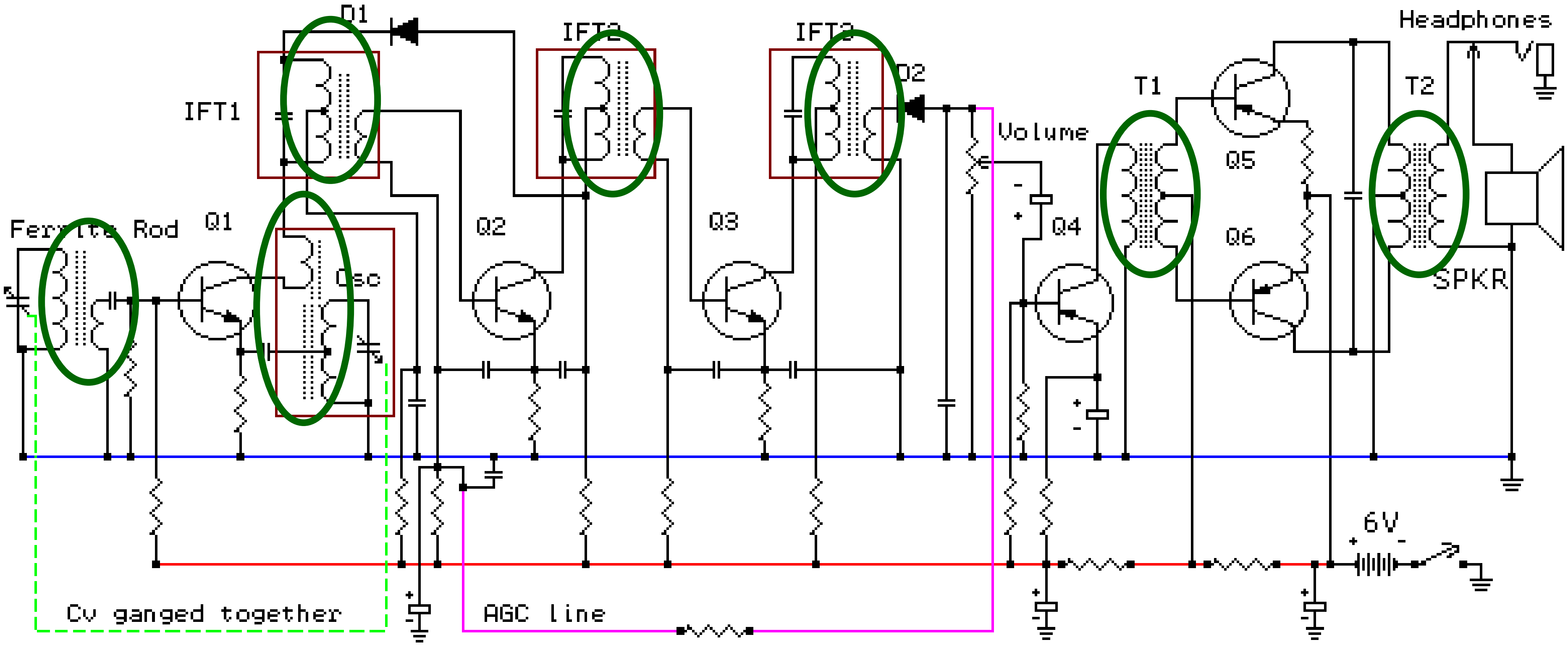


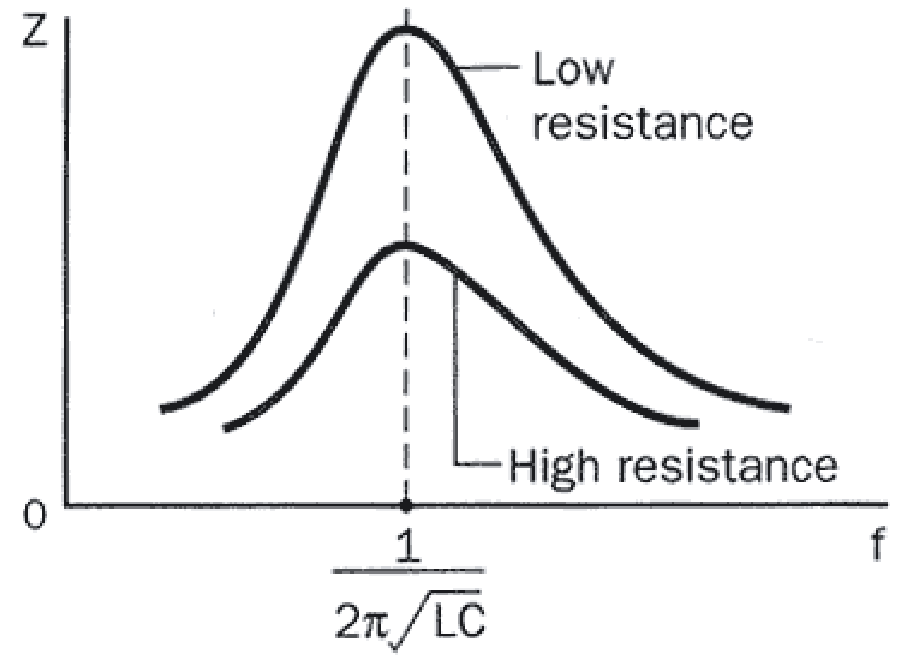
Tuned Circuits

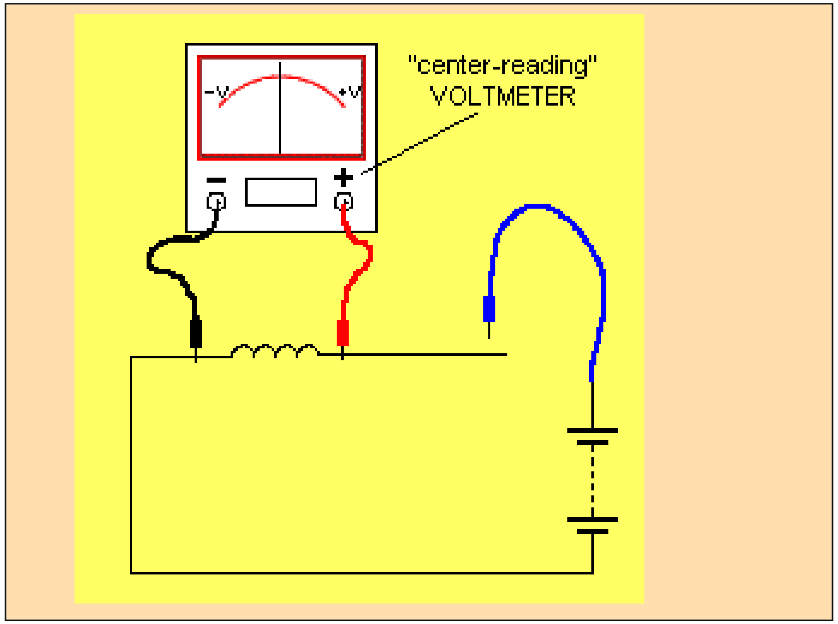
6 Transistor Radio

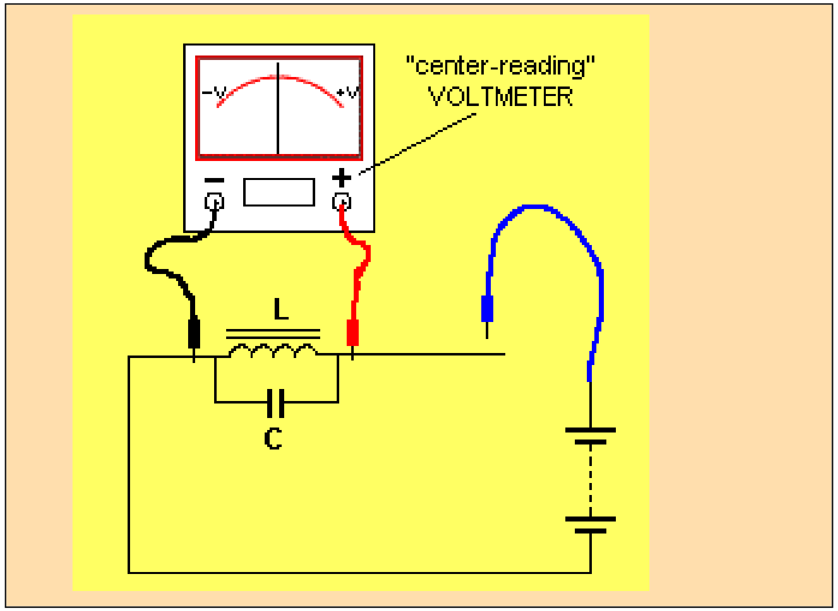


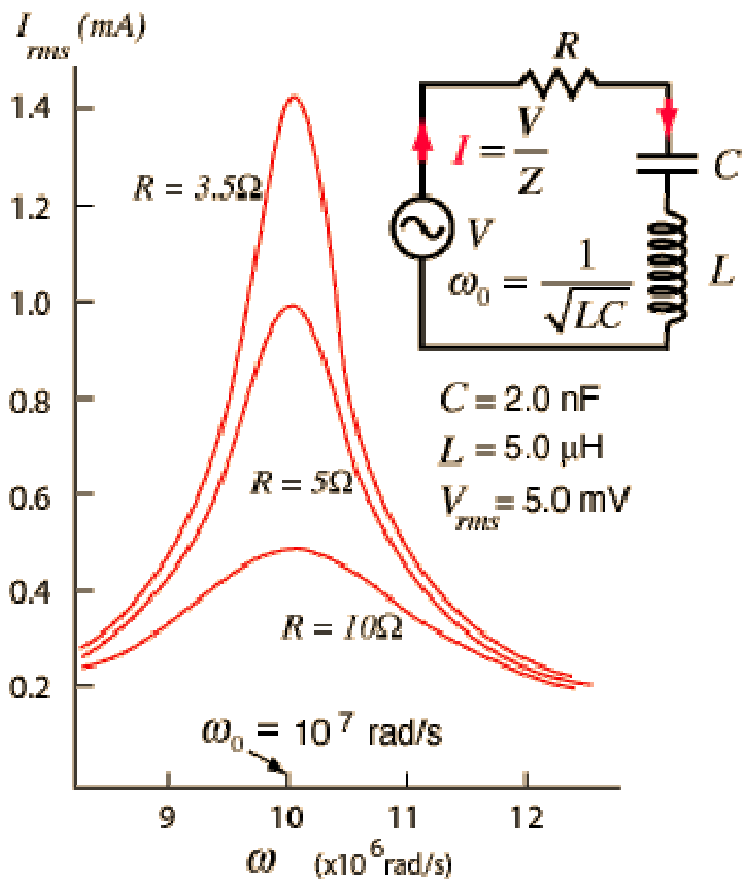
Transformers


Questions?

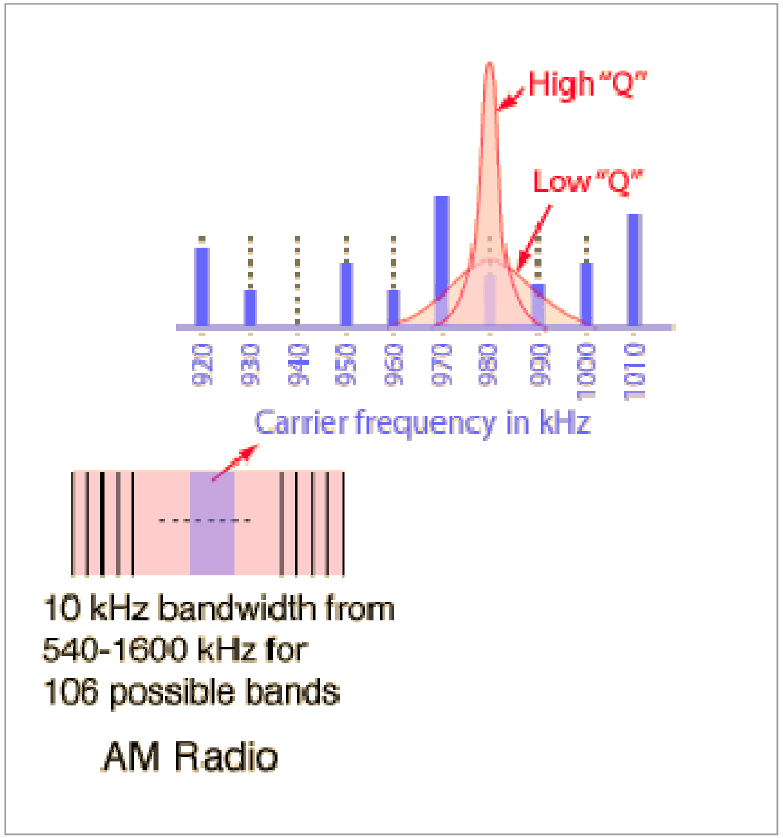

Z
Low
resistance
High resistance
0
f
1
2π√LC


"center-reading"
VOLTMETER
-V
+V
+
-

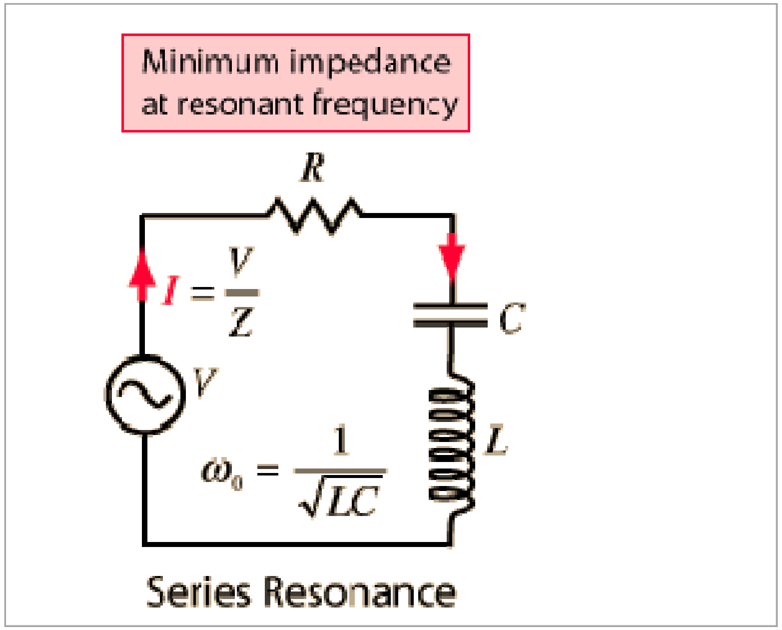

"center-reading"
VOLTMETER
-V
+V
L
C

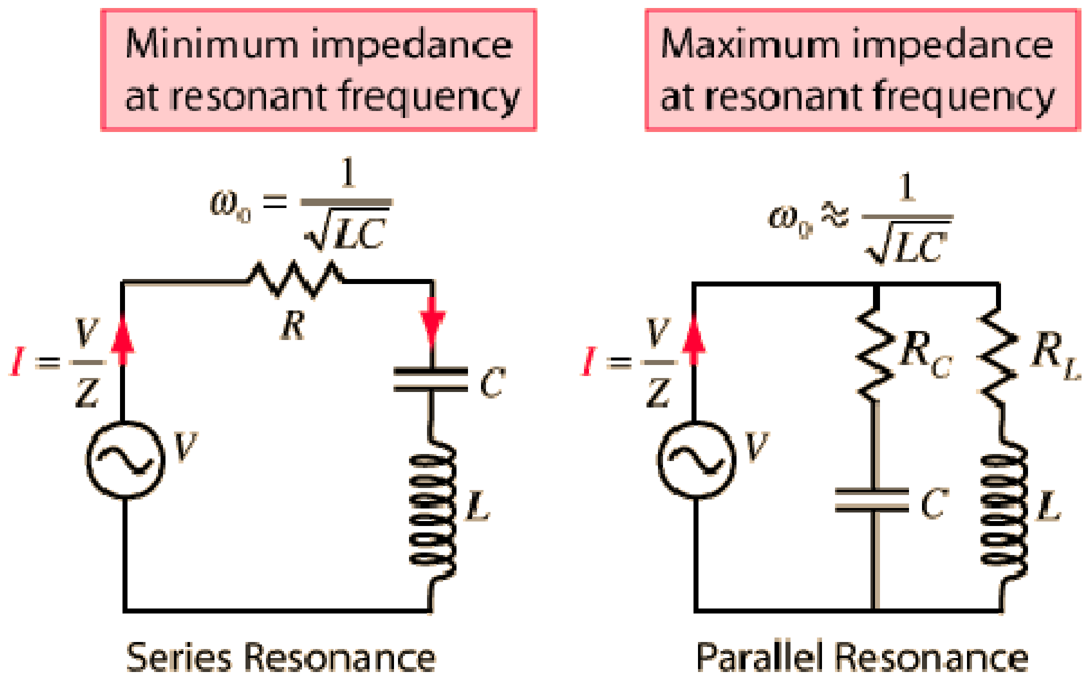


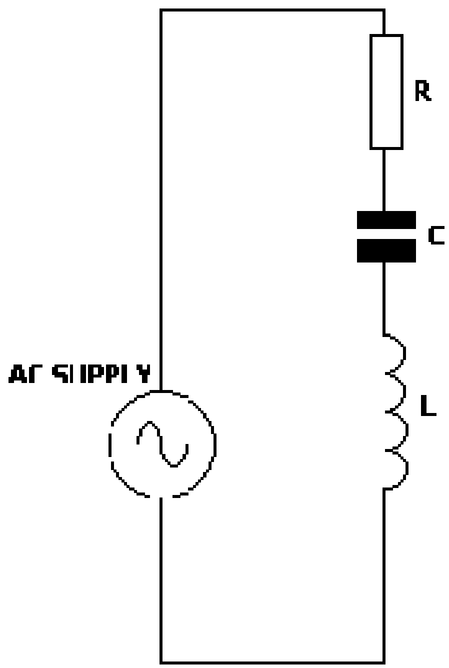
I rms (mA)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
R = 3.5Ω
R = 5Ω
R = 10Ω
R
I = V/Z
V
C
L
ω0 = 1/√LC
C = 2.0 nF
L = 5.0 μH
V rms = 5.0 mV
ω0 = 10^7 rad/s
9
10
11
12
ω (x10^6 rad/s)

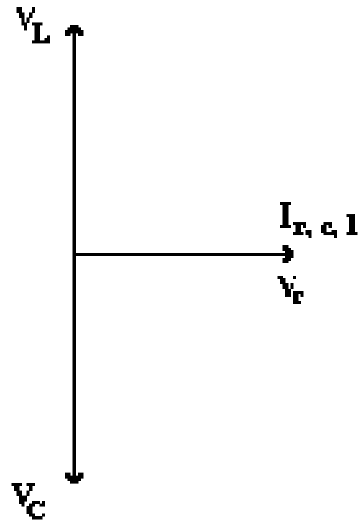


High "Q"
Low "Q"
920
930
940
950
960
970
980
990
1000
1010
Carrier frequency in kHz
10 kHz bandwidth from
540-1600 kHz for
106 possible bands
AM Radio

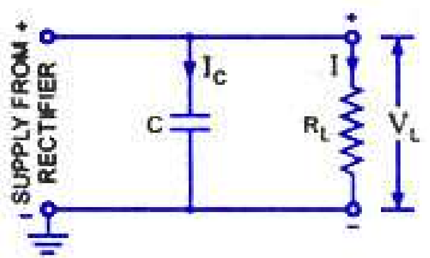

Minimum impedance at resonant frequency
R
C
L
V
$I = \frac{V}{Z}$
$\omega_0 = \frac{1}{\sqrt{LC}}$
Series Resonance


Minimum impedance at resonant frequency
$\omega_0 = \frac{1}{\sqrt{LC}}$
$I = \frac{V}{Z}$
V
R
C
L
Series Resonance
Maximum impedance at resonant frequency
$\omega_0 \approx \frac{1}{\sqrt{LC}}$
$I = \frac{V}{Z}$
V
R_C
R_L
C
L
Parallel Resonance

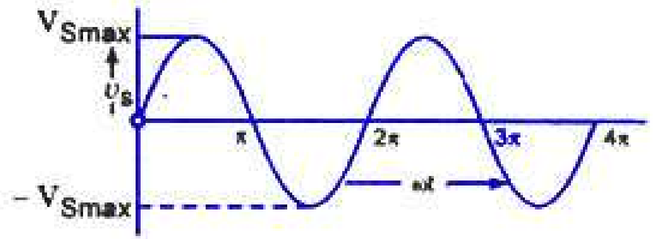




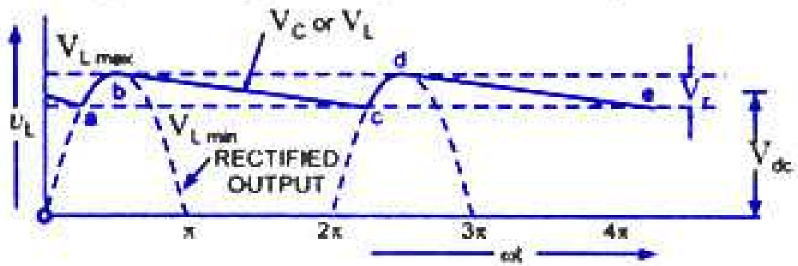
CIVIL



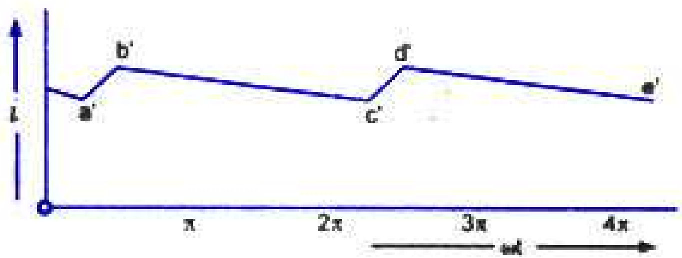
Circuit Diagram



Input Voltage Waveform To Rectifier

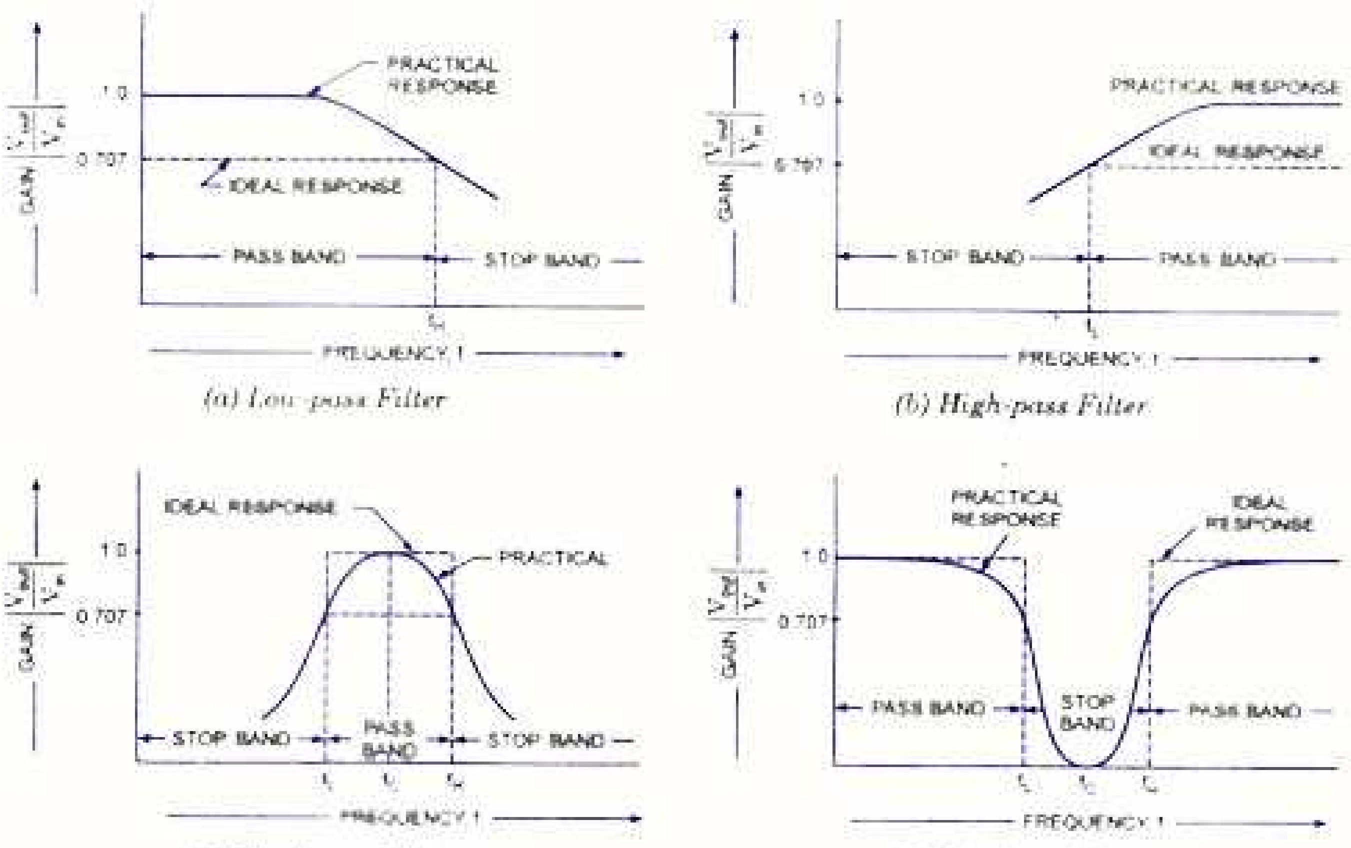


Rectified and Filtered Output Voltage Waveforms



Load Current Waveform

Half-wave Rectifier With Shunt Capacitor Filter



(a) Low-pass Filter

(b) High-pass Filter

(c) Band-pass Filter

(d) Band-stop Filter

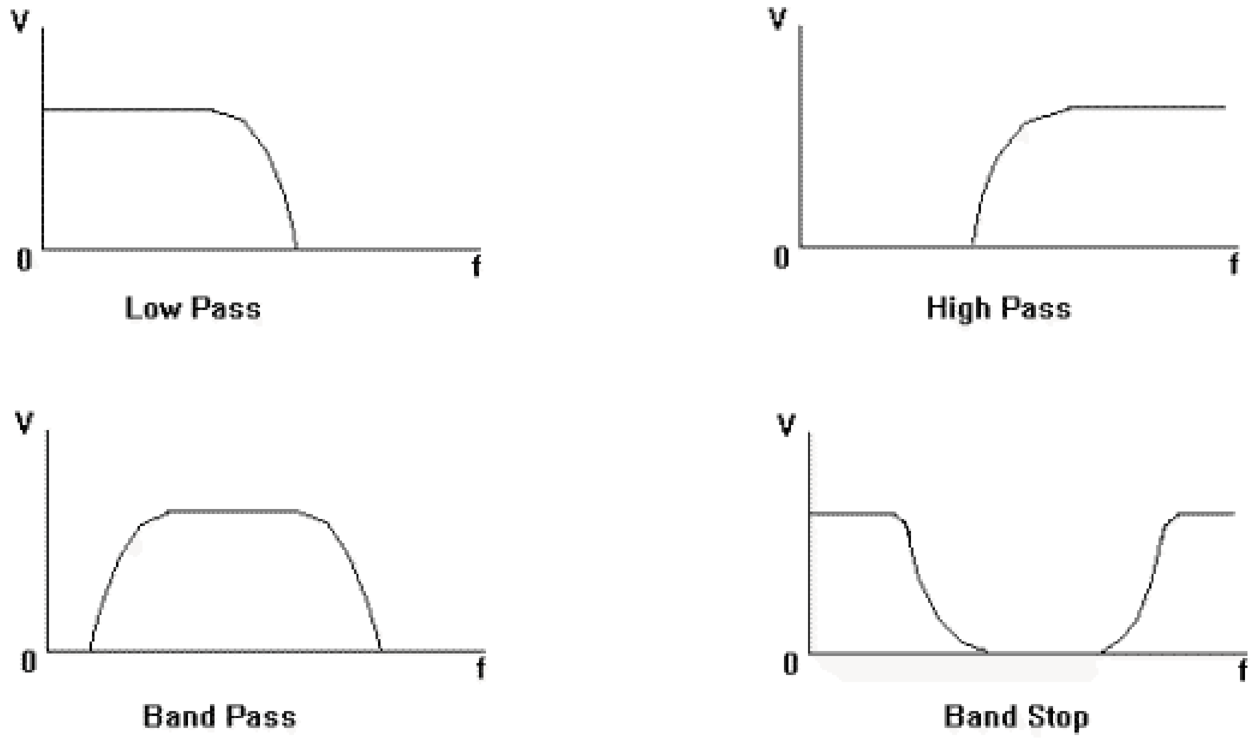
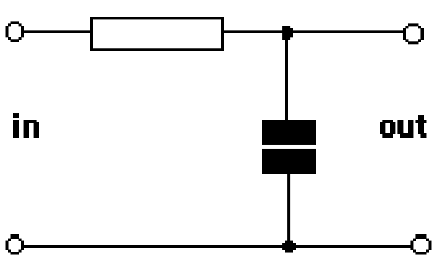
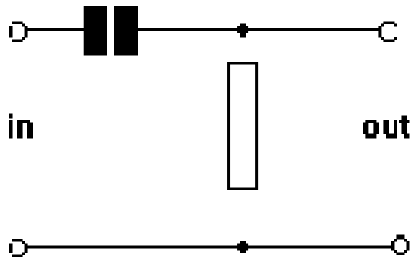


Fig. 3



Low pass filter



High pass filter

Fig. 4